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On the Import of Constraints in Complex Dynamical Systems

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Abstract Complexity arises from interaction dynamics, but its forms are co-determined by the operative constraints within which the dynamics are expressed. The basic interaction dynamics underlying complex systems is mostly well understood. The formation and operation of constraints is often not, and oftener under appreciated. The attempt to reduce constraints to basic interaction fails in key cases. The overall aim of this paper is to highlight the key role played by constraints in shaping the field of complex systems. Following an introduction to constraints (Sect. 1), the paper develops the roles of constraints in specifying forms of complexity (Sect. 2) and illustrates the roles of constraints in formulating the fundamental challenges to understanding posed by complex systems (Sect. 3).

Keywords Complexity · Complex systems · Dynamical constraints · Lagrangian dynamics · Organization · Dynamical understanding

1 Constraints (An Uneasy Introduction)

A constraint, as the name implies, specifies some limit on independent behaviours. A marble rolling in a bowl, e.g., is confined to the surface of the bowl, so that its position in any one spatial dimension is not independent of its positions in the other two dimensions. In contrast, a small spacecraft has no such constraints on its translational movement. Its *degrees of freedom* include the abilities to move in all three dimensions independently. (It can also rotate independently around an axis in each dimension; so, assuming it is a rigid body, all told it has 6 degrees of freedom.) The marble's bowl leaves it only 2 translational degrees of freedom. The bowl constraint is conveniently expressed by the relationship among positions that describe the bowl's surface. (If it is a spherical bowl of radius r then the constraint relation among the positions x, y, z is $x^2 + y^2 + z^2 = r^2$). This characterisation generalises: a *constraint* on a dynamical process is a reduction of its underlying degrees of freedom arising from the

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physical conditions in which the process takes place. A system's effective degrees of freedom are those provided by its inherent variabilities (its dynamical variables) minus those removed through constraints.[1] Constraints are expressed as relationships among system variables.

Constraints play a central role in dynamics. Currently, we work from a general Lagrangian/Hamiltonian dynamical formalism as mathematical paradigm, where dynamical motions are represented as passages along system trajectories (flows) measured (parameterised) by time and driven by energy gradients (forces) under the very general formula '$F = ma$'.[2] Typically, the forces are the basic interaction forces operating among basic system components, e.g. gravitational or electromagnetic forces among particles or chemical forces among molecules. If this exhausts the system, its behaviour is said to be free, not under constraint, that is, it has available all of its degrees of freedom, = all of the mutually independent kinds of basic motion its interaction structure permits (translations, rotations, etc.). Constraints then represent additional forces on the system that also contribute to shaping the system dynamical behaviour, but for one reason or another are not specified in detail. Typically this is because they change only on sufficiently longer timescales than the interaction phenomena we wish to study that they can be treated as constants and in that sense external to the dynamics to be studied. In any case, conveniently, their effects for the interaction dynamics to be studied can be sufficiently summarised as the requirement that certain relationships among the interaction variables be maintained invariant across the interaction dynamics.[3]

Given the glorious scientific history of our mastery of fundamental dynamics, here is a remarkable thing: within the general Lagrangian/Hamiltonian dynamical formalism we currently only know how to systematically construct coherent dynamical models for systems that are free or at least that do no work on their constraints, the canonical form for reversible dynamics.[4] If the marble rolls without friction on the bowl's surface, making contact but no more, then this last condition is satisfied. It means that the force exerted by the bowl on the marble is everywhere orthogonal to (that is, perpendicular to) its motion. This is how we normally imagine the bowl situation, but that involves assuming that gravity is the only basic interaction force and the marble and bowl surfaces are each smooth and rigid constraints; then it is easy to suppose that the marble rolls smoothly and reversibly. But suppose instead that the bowl surface is rough and deformable and so there was rolling friction between marble and bowl. Under that constraint the marble loses energy as it does work against friction, generating an irreversible dynamics of different form to that of smooth rolling. A plastic bowl might even soften when heated by the rolling friction, which would alter the constraint itself as the very motion proceeds to which the constraint gave rise, like the banks of a river eroding as the flow they channel wears against them, a radical dynamical irreversibility. Here

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[1] It is possible in principle to also constrain a system's parameter ranges—in effect, to constrain the range of quantitative forms its dynamics can take. The net effect of such constraints will still appear as (indirect, often extremely complex) constraints on variables.

[2] For constant m. According to Coleman and Korté (1999) a 2nd order differential equation of this general form is a structural feature of any ('non-pathological') space-time geometry and force field.

[3] In Sect. 3 we will return to the internal/external distinction when we inquire about boundaries and the 'self' in self-organisation—see note 31 and text, cf. Bénard cell formation and internal constraints below and note 14.

[4] Such constraints are referred to as 'ideal' constraints—meaning both that they permit the use of an elegant and powerful mathematical apparatus, Lagrangian/Hamiltonian dynamics, and that it takes its simplest form under these conditions. This last is something of a hopeful wish, as we will see—there may be no extended, more complicated but unified version of which the present formulation is a special, idealised case. Thus in this single use of 'ideal' does science run together two utterly distinct notions, ideal = best in usefulness (value), ideal = idealised (simplified). On the importance of understanding the latter sense of idealisation in complex systems see Hooker (2004, 2011c).

is where the notion of constraint starts to introduce complications to formulating dynamics. Unlike the unvarying general schema of motions and interaction forces that frame a general theory of dynamics, classically ‘F = ma’, it is a detailed empirical matter what kind of constraints there are and what conditions (like orthogonality) they satisfy, and these differences can make a difference to the basic form of the resulting dynamics, not just to its details.

As well as their motions, each of these different cases makes a distinctive difference to the nature of the mathematical problem posed by their dynamical description. If the constraints are holonomic—roughly, of fixed geometry, like the bowl surface when there is frictionless rolling—then the mathematical equations are analytically solvable and the global motion of the system can be summed up in mathematical functions (the scientific ‘gold standard’). But holonomicity easily fails, e.g. any object changing its constraint type (a ball rolling off a table edge, a flowing river) renders the overall motion constraint non-holonomic. But where work done on constraints actually alters them as motion progresses (e.g. river banks) there may be no mathematical formulation of the problem available within the current dynamical Lagrangian/Hamiltonian formalism. This grand lacuna is potentially vast, applying not only for plastic bowls and rivers but also for all living things, since in a group of self-regenerating processes (e.g. cell metabolism) the constraints that produce any one product must be largely constituted by other such products and work must be done to recreate them following their metabolic use and entropic decay (see Sect. 3 below).

The little ‘toy’ system of bowl + marble is useful for introducing another role for constraints, namely in characterising dynamical bifurcations. Suppose our bowl to be hemispherical, made of a deformable but tough plastic, and sitting evenly on a base equipped with a vertical screw at its bottom point. With the screw not touching the bowl the marble rolls around the bowl’s interior in smooth loops at all sorts of angles depending on the marble’s initial ‘launch conditions’. Now screw in the screw until a small mound appears in the bottom of the bowl. At this point any marble paths passing sufficiently near the bottom of the bowl will deviate from their old paths as they encounter the mound, altering the dynamics in a subtle but profound way that depends on the shape of the mound, e.g. the marble may now temporarily bounce off the bowl surface. As the screw is extended the effect will become more pronounced and widespread. The shift from the original dynamics to the new one is a bifurcation because it represents not only a change in specific behaviours (in trajectory shapes) but also a change in dynamical form: a differently structured dynamical equation is required to model the behaviours and the pattern of all possible trajectories (the flow) changes. This change is brought about by a change in constraints, namely in the shape of the bowl.

This is the simplest birfurcation, where the dynamical form depends on a constraint determined by the value of a single parameter, in this case screw height, and there is a threshold (here 0 screw height) at which the system changes its dynamical form and flow. A related bifurcation is the formation of ‘rolling boiling’ convection cells to replace horizontally layered conduction (Bénard cell formation) in a fluid heated from below as the applied heat flow (the relevant constraint here) passes a hydrodynamic threshold.[5] The particular pattern of convection cells obtained depends on another constraint: the shape of the fluid container. There are many other kinds of bifurcations but all depend on changes of constraint in these kinds of ways. We can be sure that bifurcations are objective and, e.g., not just mere shifts of pattern (cf. shadow plays), because new kinds of work are done, there is a shift in the dynamical form and thus energy flows.

The Bénard cell bifurcation has a feature common to most bifurcations that the simpler marble bifurcation lacks: the formation of a new internal constraint: the cells of rotating

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[5] See e.g. http://en.wikipedia.org/wiki/Rayleigh-Bénard_convection, Bishop (2008).

fluid to which fluid molecules are constrained. Creating these involves a contribution from the liquid molecules themselves, that is, from within the system. For this reason bifurcations of the Bénard cell formation kind are often called self-organising. In the Bénard cell case, e.g., fluid flows along adjoining cell walls must move in the same direction, otherwise fluid flows would collide there, destroying the cell formation; but continuity requires that the flows on opposite sides of each cell must be in opposite directions, thus cell circulations must alternate clockwise/counter-clockwise across the whole fluid. Change one cell's circulation and every other cellular circulation must also change. For this reason the cellular formation is said to show global or generalised rigidity. If this seems an active internal ordering by the system itself, it equally raises the question of how it was achieved. How is this global information generated, disseminated and effectively applied throughout the system? We lack detailed answers beyond gesturing at the idealisation of a sea of competing coherent micro fluctuations one of which was amplified by the dynamics and propagated to global fixation.

This issue of the system 'self' in self-organisation is complemented by another, the shift of constraints through an enlarging system self. The role of an apparent self arises for Bénard cell formation because the internal motions change holistically at the bifurcation threshold, in ways shaped by the container constraint. But why not absorb the container into an enlarged system as a molecular lattice and consider the bifurcation dynamics of this enlarged molecular system? Should not it reproduce the cell formation bifurcation without the need for an explicit external container constraint? This may seem 'unnatural' when the usual always-rigid metal container is involved, but it looks rather more 'natural' if the container can soften under heating so that it undergoes a mutual development in interaction with the fluid, like the river banks earlier, and even necessary if the container can melt and join the original fluid in a joint fluid dynamics. (Anyway, ultimately such intuitions are irrelevant, only the coherent dynamical possibilities matter.) And why not then also absorb sufficient of the heat source as well, so that both erstwhile constraints are absorbed into internal dynamics? And on to absorb the dominant source of gravitation and so on until the cosmos be encompassed. Can we not in this way do away with constraints altogether?

Certainly, there are many cases of no-external-constraint dynamics that show complex behaviour, e.g. the solar system (including a moving sun) can shift from coherent into chaotic motions. But here no external constraints were ever needed. As soon as we consider other systems, the constraints re-emerge, only now as internally generated. In both the usual and enlarged system representations the Bénard cell container's metallic molecular lattice must still constitute a constraint on the motions of its fluid molecules, sufficient constraint to retain the boiling liquid within them and impel a cell shape and a horizontal arrangement of cells. Constraints do not have to be pre-formed: free-falling liquid raindrops form internal molecular lattice constraints as they cool that join to form snow flakes, a lattice that thereafter constrains all their molecular motions, and this holds for all cooling phase changes. And so on. The point is, constraints are as dynamically real as the focal interaction dynamics of a system, because the constraints themselves are generated from the interaction dynamics of the elements that subserve (are the sources of) those constraints and so is their capacity to filter out the thus-constrained motions of the interaction dynamics. One can deduce from the properties of a metal lattice (the constraint) the existence and properties of the Fermi conduction bands (the filtered electrical conduction behaviour). The internal formation of constraints is the most dynamically robust form of emergence since a new dynamical individual is objectively born from the bifurcation, and thus carries new causal (energetic) powers. So external constraints cannot be made to disappear; instead they re-appear as internal constraints, where they also stand at the centre of understanding self-organisation and selfhood.

A final feature of constraints will complete this introduction to the idea. The term 'constraint' implies limitation, and specifically here it refers to limited access to dynamical states or, equivalently, reducing degrees of freedom by limiting dynamical trajectories to sub-sets of the basic interaction state space. This is the common *dis*abling sense of the term. But constraints can at the same time also be *en*abling, they can provide access to new states unavailable to the unconstrained system: equivalently, by coordinatedly decreasing degrees of freedom they provide access to dynamical trajectories inaccessible to the unconstrained system. For instance, low resistence electrical conduction is a state available to a metal lattice but unavailable to the metal molecules as a collection of independent entities (e.g. in a gaseous state). More generally, a skeleton is a disabling constraint, for example limiting the movements of limbs (cf. an octopus), but by providing a jointed frame of rigid components for muscular attachments it also acts to enable a huge range of articulated motions and leverages, transforming an organism's accessible niche, initiating armour and predator/prey races, and so on. It was Pattee who emphasised the importance of constraints, especially of such coordinated constraints, to biological organisation and evolution.[6]

Each of the eight great transitions in evolutionary history (Maynard-Smith and Szathmary 1995), e.g. the emergence of multi-cellular organisms, marks a new coordination of constraints. By permitting reliable cooperation instead of competition and reliable inheritance of the fruits of cooperation, these coordinations created new complexity and opened up vast new possibilities. Moreover, the constraints simultaneously operative in biological systems (a) are typically mechanical and chemical, solid and liquid, and so forth, (b) operate in an equal diversity of ways, e.g. in development they include quasi-equilibrium effects (differential adhesion), non-linear and non-equilibrium processes (chemical oscillation), phase transitions (epithelial-mesenchymal transformation; skeletogenesis), etc., and (c) operate across a variety of levels and spatio-temporal scales (Newman 2011a,b). Coordinated constraints can work their way around physical laws, e.g. while no single pump can lift water higher than 10 metres, trees lift it several times this by linking together many cellular pumps. While it is possible to obtain complex dynamics in simple systems, plausibly the only way in which the complex properties we will shortly introduce can be obtained is through complex coordination of constraints of the kind neural, muscular and skeletal coordinations exemplify. These begin already with the complex coordination of biochemical products and gradients that allow intra-cellular chemistry to support cellular maintenance. We are here far from the holonomic constraints of standard analytical mechanics and deep into the domain of multiple state-dependent, interacting, non-holonomic constraints.

Constraints, external or internal, specify context, the dynamical context in which interaction dynamics take place. But in this role constraints often do not stand aloof from interaction dynamics, as they do when in holonomic form, but rather they so interrelate with the interaction dynamics as to enable new dynamical forms and with that new selves (cf. note 39 on contextual emergence and reduction). Context and content are intimately interwoven, the most basic expression of the antinomy of individuality and wholeness. In Sect. 3 we will inquire a little further both into the notion of 'self' (note 3) and that of 'organisation', each so

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[6] See Pattee (1971, 1973). Pattee (1973) also draws attention to the way in which constraints are typically specified as macroscopic effects relative to the dynamical relations involved in their constitution and often also relative to the specification level of the interaction dynamics, thereby implicitly defining the locus of a partition between macro and micro, a partition that could potentially be moved to encompass more and less within the micro level specification. This is, he suggests, directly reminiscent of the 'cut' between (relatively) macro observer and micro system in quantum mechanics. While this is intriguing, Pattee rightly remarks that after years of discussion the latter 'cut' is still not satisfactorily understood and, if my argument of Sect. 3 holds, neither is the partition for systems that alter their constraints by doing work on them.

glibly used in ‘self-organisation’, and into the equivalence of internal and external constraints discussed here.

2 Complexity (A Constrained Review)

With some feel for the notion of constraints in complex systems, it is time to turn to the first objective, to spell out the role of constraints in specifying forms of complexity, many of which tend to be overlooked. The preceding discussion has already thrown up several examples of complex behaviours, e.g. bifurcation thresholds and domains, and self-organised emergence through constraint formation. Unhappily, there is currently no unified theory available that captures all the cases of complexity, neither a fundamental dynamical one nor some higher order characterisation. We simply don’t know enough, especially about the relevant mathematics, to tackle such an ambitious goal. Here the aim instead is to illuminate the principal system properties involved in complex dynamics, in whatever combinations, with particular attention to the roles constraints play in their conception and operation.[7]

Group 1: *general dynamical properties*. The discussion begins with a group of general dynamical properties that underpin later, more specialised properties. Here basic interaction dynamics dominate their characterisation and constraints play either no role or simply play their general dynamical role. There is, it would seem, just one general property that is necessary to obtain any form of complexity: *non-linearity* of basic interactions and the consequent *failure of additivity* (*non-superposition*).[8] Other properties that might have been considered contenders for necessity, especially *non-holonomic constraints* and *irreversibility*,[9] prove not necessary to complexity, as the example of unconstrained reversible 3-body gravitational systems shows. Neither are any of these properties sufficient for complex behaviour, as respectively shown by the examples of 2-body gravitational motion for non-linearity, a contained gas for non-holonomicity, and a freely expanding gas for irreversibility. We currently have no general account of which kinds and combinations of these properties will yield which classes of complex systems. However, as we approach what are intuitively more complex systems all three properties tend to form a common basis, e.g. for all living systems.

Neat formal criteria for complexity wont help us out of the woods here. The most prominent proposal is that of *algorithmic complexity*, roughly measured by how compressed (shortened) a description can adequately replace a complete detailed description of a system. But while repetitive systems like lattices come out simple, it leaves random gases, that generate no interesting complex behaviour, as hugely complex (since every molecule has to be separately

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[7] For a more general discussion of these properties the reader is referred to Hooker (2011b,d), from which parts of this discussion, modified and improved, have been adapted. One advantage to setting out the extended set of complexity-making properties is avoiding a current tendency toward the mis-placed simplicity of focusing on just a select few characteristics, e.g. just chaos, or counter-intuitive/ unpredictable behaviour, or having a long incompressible description (algorithmic complexity).

[8] An interaction is *linear* in some variable v if the interaction force varies proportionately to v (formally, $F(kx) = kF(x)$, k a number) and non-linear otherwise. Gravitational force, e.g., is spatially non-linear since it varies as the inverse square of the interaction distance ($G(kr) = k^{-2}G(r)$). Interaction linearity yields linear dynamical equations describing the system and these are characterised by additivity: any numerical sum of solutions is also a solution. No complex dynamical behaviour is possible.

[9] A constraint is *holonomic* if its local (momentary, nearby) differential form is integrable to yield a global constraint relation that is purely a matter of space-time geometry and independent of the system’s dynamical states, e.g. the frictionless bowl as constraint for the rolling marble, and otherwise is non-holonomic (cf. note 37). A process is reversible if its dynamics (its state sequence structure) remains the same under time reversal (and only the time-ordering of the sequence structure is reversed), e.g. if it obeys Newton’s laws; and otherwise is irreversible. Irreversible processes involve energy dissipation.

specified) while blurring over all the distinctions in between (see below). The most complex systems described below, organisms, will be of large but intermediate value in algorithmic complexity.

Our next property is *complicatedness of dynamical domains*. The marble in the bowl can roll through the centre of its bowl or not; if it does the motion is an oscillation in a straight line while if not it consists of a loop about the centre of some kind. This yields two dynamical domains. A complex dynamics may generate several different equilibria of various stabilities (attractor basins of various shapes) either directly intersecting, even having fractal common boundaries, or connected by transient paths (where small disturbances may change its destination, cf. rolling on horizontal surfaces) plus other transient paths that do not end.[10] This ‘attractor landscape’ is a system’s dynamical signature, expressing its dynamical form. By contrast, a linear dynamics has no attractor basins, hence no equilibria, simply a ‘flat’ landscape filled with transients. A system that remains within a single attractor landscape is structurally stable (= autonomous dynamics in mathematical parlance) and otherwise is structurally meta- or un- stable. While mathematical dynamics typically assumes structural stability, many complex systems are structurally unstable, e.g. exhibiting phase changes. Inter-landscape transitions are bifurcations. All this invites introducing both complicatedness of attractor landscape and of inter-landscape transformations as aspects of dynamical system complexity. In both cases constraints combine with interaction dynamics to create these complexity features in the manners already illustrated.[11] In principle, all of the properties to follow correspond to features of landscapes and landscape transitions; but we are far from being able to actually construct such systematic correspondences so it is still most useful to discuss particular properties.[12] Anyway, the roles of constraints are implicitly enfolded into the resulting landscapes rather than being made explicit as is the aim here.

There follow a collection of properties common to many complex systems but which are noted only in passing here since they do not concern new roles for constraints. *Amplification* locally and up to whole system level, especially the amplification of small perturbations and perturbations localised to a few components, is common in non-linear systems. Critical point birfurcations and many others where a new system constraint is formed are thought to occur through amplification of a fluctuation at component level in this way. Similarly,

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[10] Equilibria of any sort are stable, meta-stable or unstable. An equilibrium in some aspect A is stable, with respect to some class of disturbances (perturbations) D, if (and only if) its response to any disturbance from D is to soon return near to (including exactly to) its original A condition under its own dynamical processes and remain there. An equilibrium is unstable to a class D of disturbances if it does not return near to its original A condition and it is meta-stable to D if it is stable for some disturbances from D and unstable for others. This formulation holds for both static and dynamical equilibria, it is just that in the static case it applies to a set of state parameters and variables while in the dynamical case it will apply to a set of process parameters and rate variables. The closed set of states a system repeatedly traverses when at equilibrium is its attractor (the marble’s rest point is a point attractor, if it circled frictionlessly around that point but up the basin wall it would be a cyclic attractor) and the wider set of states it can transiently pass through while still returning to its attractor is its ‘attractor basin’ (the bowl provides a literal attractor basin).

[11] Scientists will often speak of stability as itself a constraint. This is potentially confusing since, read literally, the claim is a category error, stabilities are facts characterising a system dynamics in some domain, they cannot of themselves act as constraints within that dynamics. Instead scientists typically intend only the metaphorical claim that unless a system’s dynamics is stable it will cease to retain its identity as that system, or perhaps the related metaphor that some system has been selected for its stability under certain conditions, so stability is among the selection constraints; in either case it is just a way of claiming that stability is a materially necessary condition for system existence.

[12] I am using the term ‘complicatedness’ here, not as a question-begging substitute for ‘complexity’, but simply as a stop-gap for some specific measure or other of numbers, kinds and connectednesses of attractor basins and landscapes. I doubt there is any one clearly preferable measure and anyway actual measures cannot at present be more than very partially applied (cf. e.g., Wackerbauer et al. 1994).

every inter-landscape shift corresponds to *symmetry-breaking*. The change from conduction to convection in the formation of Bénard cells, e.g., corresponds to the breakdown of previous conductive horizontal symmetry; in consequence the orderedness and constraint coordination increased, delivering an increase in complexity of the system behaviour, and this is typical.

Amplification is where small differences in system state are amplified into large differences in subsequent system trajectory; this is *sensitivity to initial conditions*. Under certain conditions it takes a special form where a *strange attractor* is formed in the attractor landscape in which there is said to be *chaotic motion*. However, the motion remains deterministic and, far from being more disordered than a normal attractor, is best viewed as super-ordered since every point within it may manifest sensitivity to initial conditions. Systems manifesting sensitivity to initial conditions present the problem that small uncertainties (including errors) in knowledge of initial conditions may be amplified into large subsequent uncertainties (and errors) in system state. This yields *limited predictiveness* because system predictability is limited by limited knowledge of initial conditions (even when dynamical form is fully known). How severe a limitation this is in practice, and in what respects, depends on the amplification processes and uncertainties/errors involved. For instance, while prediction that a system's state will remain within a strange attractor is often legitimate, knowledge of location within the attractor can be quickly lost.[13] However, it is also true that systems showing sensitivity to initial conditions can be significantly influenced using only small signals (as perturbations to be amplified). The ideal condition for this behavioural sensitivity is near to the chaotic condition, where there are multiple sensitivities, but not fully chaotic (in a strange attractor) where there are too many of them. This is *edge-of-chaos criticality*, a condition toward which many complex systems move, from sand piles to sensory neural detectors.

Group 2: *Constraint altering dynamical properties*. We come next to two properties, already briefly introduced, where the dynamics of constraint change plays a key role in their clearest conception. *Self-organisation* occurs when, through existing constraint coordination, a system bifurcates, sufficiently under its own dynamics, to a form admitting at least one new constraint or (equivalently) new constraint coordination—for instance, in Bénard convection cell formation. There will thus also be new behaviours, sometimes more complex (e.g. Bénard cell formation) and sometimes less complex (e.g. crystal lattice formation). By contrast a reverse bifurcation, where those constraints are instead lost, though also a dynamical landscape transition, would not normally be considered a self-organisation. (It might be considered a self-disorganisation). Since the condensing of molten iron to form a solid iron crystal is also considered self-organisation it is clear that self-organisation has little to do with organisation proper, since an iron crystal is too ordered to be significantly organised (see below).[14]

Emergence: When the system outcome of dynamical interaction among system components is surprising or unexpected or too complex to readily understand, scientists are apt to talk about emergent patterns, but this is a vague, shifting and subjective approach. Tightening

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[13] But for some of the surprising subtleties surrounding this and like notions, see e.g. Bishop (2011).

[14] All things considered, it would make for clearer communication if 'self-organisation', when used generally, were replaced by 'self-ordering' and 'self-organisation' used according to the definition in the text. Note that what counts as "sufficiently under its own dynamics" to justify the 'self' in self-organisation can be a vexed matter. The pan of heated fluid counts as self-organised because, while the applied heat is key to 'forcing' the dynamical changes, it is applied sufficiently externally to the fluid dynamics. If we deliberately add species to an ecology until it achieves a certain resilience to drought, on the other hand, it would probably not be considered to *self*-organise that dynamical form transition. The system could also not be considered to simply have been organised either, because the outcome may not be increased organisation (though, confusingly, we could say that we 'organised' the change—see note 25). See further Sect. 3 below, note 31 and text, and Hooker (2011d).

it to the appearance of a phenomenon that could not have been predicted from knowing only the pair-wise dynamical interactions of components, is sharper, but still ties the definition of evidently physical properties to a cognitive test (prediction) and, since prediction is so limited, far too much would count as emergent. A better option is (as always) to pursue a dynamical criterion. A clear, wide criterion would be to identify emergence with bifurcation generally, a clear narrower one would be to identify it with just self-organisation. In each case a new dynamical landscape forms or comes into being (intuitively: emerges) through constraint change. On the other hand in the bowl+screw case no new kind of physical work is done; the constraint merely changes its geometric detail. In the self-organisation cases, by contrast, a new existent doing new physical work emerges in consequence of a new constraint emerging.[15] The wider criterion would include some fully explainable cases (e.g. the bowl+screw bifurcation), the narrower criterion would not (see Sect. 3). There do not seem to be any other interesting, clear definitions (cf. Hooker 2011d).

Two further properties can now be characterised. *Path-dependence* is the consequence of positive amplification since then initially nearby dynamical trajectories subsequently diverge as a function of small differences in their initial conditions, so the path taken depends on precisely where the first step began. A notable sub-class of path-dependencies are those where, once begun, development along a certain path itself becomes reinforced, e.g. where an initial fluctuation is amplified and entrenched, especially where that entrenchment involves a bifurcation that makes the development irreversible. Examples include a particular impurity site of first freezing or rolling boiling; a first stage in a developmental trajectory that makes possible a further, perhaps more important, stage; a first genetic mutation that yields a distinctive kind of adaptive advantage; a first oil discovery or shop in a new suburb that transforms a local economy. *Historicity*. These cases also exhibit clear senses of historical possibilities taken up or foregone and their resulting paths are often said to ‘fix’ their initial historical conditions.[16] By contrast, for stable systems in an attractor basin there is no overall path-dependence since the same outcome occurs (capture by the attractor) for all beginning points (initial conditions) in the basin. Developmental historicity plays an essential role in biology, e.g. whenever there is an earlier stage in a developmental trajectory that makes possible a further, sometimes functionally more important, later stage (Newman 2011a,b). A characteristic historicity to the internal make-up and behaviour of a system, e.g. for an organism, expresses its individuality and so strengthens its sense of self.

And with (relatively) macro constraint formation providing a principled notion of system level,[17] two further properties follow. *Modularity* obtains when system constraint coordination is such that system dynamics can, to a sufficiently good approximation (e.g. to capture essential system functionality), be expressed as an interactive product, the dynamical product

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[15] These situations are thus said to show ‘top-down’ constraints or ‘downward’ causation, e.g. Bishop (2008), Emmeche et al. (2000), O’Connor and Wong (2002).

[16] However, it would stretch the notion of *physical* constraint to vacuity to call all these initial conditions path constraints, because there is often no cohesive force involved that could ground the constraint.

[17] ‘Level’ is a loosely (ab)used term. The formation of a new (relatively) macro constraint, however brought about, creates a new level proper in the system, since the constraint now filters out microscopic detail incompatible with it. The iron crystal lattice, e.g., filters out thermal fluctuations and many external perturbations, dissipating their energy as lattice vibrations. (Otherwise the constraint would not be stable against microscopic-originated perturbations and similar external disturbances.) The iron becomes a 2-level system, (1) that below the level of the lattice, the individual ions and electrons, obeying their dynamical interaction laws, and (2) that at the lattice level with its fermi conduction band where electrons stream through the lattice, the lattice collectively vibrates, and so on. This is a dynamically well-defined and grounded notion of ‘level’, all other uses are for gravitation (e.g. level table) and measurement (e.g. flood level) or are metaphorical (e.g. abstraction level) or confused.

of its intra-modular dynamics and its inter-modular dynamics.[18] Three kinds of modularity can be distinguished, spatial or 'horizontal' modularity at the same macro level (e.g. groups and populations, most buildings and machines), cross-level or 'vertical' modularity (e.g. caste and class social models, business managerial models, scale models of organisms as cells, organs, organism) and process modularity (e.g. models of organisms and complex machines as mechanisms, such as respiration and pulp mill regulation). *Hierarchy* proper is asymmetry of vertical constraint in a sufficiently vertically modular system. It is the exception, the commoner case being mutual constraint both upwards (components constraining their macro level, e.g. ions constraining crystal lattice angles) and downwards (macro level constraining components, e.g. lattice constraining ion vibrations). Modularity reduces system complexity, by decreasing dynamical degrees of freedom, while increasing functional and possibly developmental reliability and ease of repair, but potentially at the risk of removing more subtle but powerful higher order intra-system relationships like multiplexing and multitasking.

Group 3: *Global*[19]*functional/organisational constraints*. Here we shift to properties characterised essentially in terms of input/output relationships and only indirectly in terms of the dynamical processes that subserve those relationships. A function F: x −> y takes input x and outputs y; this is as true in mathematics as physics; but in real natural systems, where x, y are real conditions, the transform '−>' must be a real dynamical process that transforms x into y using the basic interaction dynamics and suitable constraints. These constraints, we shall see, can become crucial to the realisation of the function.

Of natural necessity the capacity to carry out functions is central to biology. First, as systems subject to the 2nd law of thermodynamics, they must take in metabolic resources (that is, food and water) and export entropically unusable end-products as wastes; this is their most basic internal functional requirement. Second, as ecological creatures, they must also do this while food-hunting and mating successfully and avoiding predation; these are their most basic ecological functional requirements. Many more functions derive from these basic ones, e.g. respiration as part of metabolism, sensory-motor stimulus-response coordination as part of functioning in an ecology. And of course functionality is not confined to living creatures, almost every artifact is designed to serve some function, from a car engine to a teapot—both of the latter being designed to convert a stored resource (chemical energy, water+tea leaves) to a valuable usable product (mechanical torque at axles, drinkable tea in cups). Another dimension of complexity is the number and subtle interrelatedness of the functions served by a system. The tea pot is a simple device, the car engine much less so.

The car engine holds the clue to the importance of functions for us here. A traditional car engine has many different parts (carbureter, pistons, valves, con-rods, tie-rods,...) with each part playing a distinctive and essential role in the whole to bring about the function. It is the coordination of these roles that yields the dynamical process that realises the function of creating output usable torque from input fuel: inserting fuel/air mix, compression, combustion, cam shaft torque, exhausting waste products. Although no constraints appear

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[18] Then all system components, at whatever degree of abstraction, are modules, with basic components being those taken to have no internal dynamics and fixed inter-modular dynamical interactions.

[19] 'Global' has been used to mean 'spanning the whole of'; thus a global constraint is one that is applied to the whole system, that spans the whole system, as opposed to constraints that apply more specifically to particular sub-systems or parts. A global constraint is ipso facto a macro constraint, while a macro constraint must have at least a global component to it. Both global and macro constraints, functions, etc. can be relativised to sub-systems or parts of systems and would be called 'relatively global/macro'. Often one wants to say something about all constraints, functions, etc., relatively global/macro or not. For ease of expression, in what follows the relativisations will simply be assumed, except where clarity requires explicitness.

explicitly in this process description, to perform their roles while preserving their exact role interrelationships, requires that the parts operate under a very specific set of constraints, in this case realised in the rigidity of the con-rods etc. and through parts whose only purpose is constraint, including the engine housing and the tie-rods, whose purpose is to lock the whole assembly together. So the global functional constraint of converting fuel to usable torque is itself realised in a carefully interrelated collection of more specific constraints. Without this specific coordination of constraints we cannot dynamically realise the global engine function.

If we turn to biological functional constraints we must find the same. For instance, respiration requires multi-level processes from intra-cellular Krebs Cycles to somatic cardio-vascular provision of oxygen and removal of carbon dioxide, processes that must be made coherent across the entire body and from sub-cellular to organ levels, while also proceeding more or less mutually independently within the coherence requirement. All this requires the intricate coordination of manifold constraints at many levels.[20] A particularly subtle version of these constraints must operate in the special case of intra-cellular function since cellular metabolism manufactures all or most of their required chemicals internally from simpler input components in such a way as to regenerate themselves, including this same self-regenerative capacity. This means that the products of some interactions act as constraints for the production of others in a network that loops on itself to achieve self-reproduction, an organised network of state-dependent, interacting work-constraint cycles (Kaufman 2000). DNA itself is a set of coordinated constraints that catalyse RNA production and RNA similarly catalyses protein production and so throughout cellular biosynthetic pathways. Mobile coordinated constraint sets of these kinds are critical to evolution and development.[21]

Such constraint networks can be anticipated to be structured so as to allow *multiplexing* (many component roles combining to realise a single function) and *multitasking* (the one component playing roles in realising many functions). Multiplexing and multitasking are attractive because they reduce the number of required components while increasing system functionality and adaptability, and possibly evolvability, thereby increasing, while adding additional dimensions to, system complexity (and possibly also increasing system instability

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[20] The overall process of respiration is *multi-level*: involving sub-cellular to organism coordination, *multi-dimensional/plexed*: involving organised interactions among many body parameters, *multi-modal/tasked*: involved in many different bodily modes of operation (motor, cognition, stress, etc.), e.g. the cardio-vascular system simultaneously transports resources (oxygen etc.), wastes (carbon dioxide etc.), regulatory hormones, and so on, and *multi-phasic* (asynchronous and non-stationary): respiratory processes occur on many different timescales, with local parameters constantly changing functions of temporary activity while more global respiratory parameters are functions of the longer term developmental and subsequent functional history of the organism. In this conception, slower, more global processes set changing constraints for faster, local processes and their relative mutual independence is achieved if the sets of micro components realising each are largely or wholly separate and those involved in the slower, more global process filter those in the faster, local process from disturbing it, except under specific functional conditions (e.g. to trigger macro actions via macro meta-stabilities). Then global coherence is a result of internal regulation at various, more or less independent, functional levels (intra- and inter- cellular, organ and body). It is processes like these that we need to come to grips with if we are to capture real biological functions, but although the basic interaction dynamics is mostly well known, we are not close to modelling the intricate process and constraint interrelations involved. For further discussion and progress in modelling see, in biology, e.g. Bechtel and Abrahamsen (2011), Newman (2011a,b), and in robotics e.g. Clarke and Proença (2009), Mackworth (2009).

[21] Bird beak development, e.g., is controlled by production of a protein BMP4 that regulates the rate of growth of the underlying mesenchyme (embryonic connective tissue), which in turn forms the species-characteristic upper and lower jaws, its production rate and timing explaining differences in beak shapes from Darwin's finches to chickens and ducks (Newman 2011a). Such mobile morphogens (Turing 1952; Newman and Bhat 2008) offer a powerful way to generate massive variety in form from small alterations in sequence and timing and may prove to be a foundation for embryogenesis (Newman 2011b), one creative enough to confound critics of evolutionary biology on grounds of its explanatory inadequacy (e.g. from so-called intelligent design).

and/or rigidity). Such organisation forces a correlative complex *multi-level spatio-temporal process organisation* (cf. respiration, note 20). Multiplexed, multitasked functions cannot all be realised simultaneously at every location, the resulting interference would render reliable performance impossible. It is necessary then to distribute the realising dynamical activities spatially and temporally so that each local area over each process cycle is restricted to a coherent set of concurrent activities. Moreover, these distributions have to be subtly organised so that each function is realised at convenient locations and times for receiving its inputs and also useful locations and times to contribute its outputs. Currently we often have only sketchy knowledge of how those coordinated constraints are physically realised.

Nonetheless they form the basic organisation characterising all life: *autonomy*, namely the coordination of the internal metabolic interaction cycle and the external environmental interaction cycle so as the latter delivers energy and material components to the organism in a usable form and at the times and locations the former requires to complete its regeneration cycles, including regeneration of the autonomy capacity.[22] And autonomous organisation in turn forms the proper basis for *adaptation*, *adaptiveness, agency* and *learning*.[23]

In sum, it is the specific structuring of coordinated constraints that is the necessary condition for utilising the available basic interaction dynamics so as to produce the dynamical processes that realise global functions and the many more specific sub-functions they in turn require.

That formulation also holds true of the possession of *functionally active boundaries*. At its most general a boundary is simply a division made so that it singles out a unique interior. Mathematics uses the notion in that general, abstract sense, e.g. in boundary value problems. Even within physical systems, some systems have no non-arbitrary physical boundaries, e.g. gravitational systems (sun, solar system, galaxy) since the gravitational force has no shield and no null locations, and this is still more common in social systems, e.g. consider the boundary of a broadcast concert audience in the presence of manifold transmission and playback technologies. This and like complications encourages the standard practice of placing a system's boundary so that it internalises all interactions capable of making a sufficient difference to system behaviour as to be evaluated as significant to the purposes at hand. More narrowly, some physical processes naturally identify boundaries within and of themselves, e.g. fluid boundary layers. While this latter takes us closer to the desired notion, it and all the preceding senses of boundary are too weak for the present notion, which is that

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[22] On autonomy see Christensen and Hooker (2000a, 2002), Bechtel (2007), Moreno (2007), Hooker (2011b), Moreno et al. (2011), and references. Self-governance lies at the core of our commonsense conception of autonomy. However, we are most familiar with the idea of autonomy as applied to persons and political governance, but these are sophisticated notions applied to sophisticated systems whose trappings may distract from fundamentals. We need to return to basic principles operating in all living systems to construct a naturalist notion that will 'grade up' across the evolutionary sequence to our sophisticated concept.

[23] An organism is adapted when it possesses an autonomy-satisfying set of traits in its life-environment. Conversely, an organism's ecological niche is comprised of the range of life-environments for which its traits provide satisfaction of autonomy. An organism's adaptiveness or adaptability is its capacity to alter its specific traits in mutually coordinated ways so as to adapt to, that is, satisfy autonomy in, a wider range of life-environments. Learning, understood most generally (e.g. for populations and ecologies), is the application of adaptability to develop adaptations; understood more narrowly (e.g. for individuals), it is this process manifest through internal sensory, memory and motor regulation rather than inherited reaction to stimuli, that is, through neurally modulated behaviour. It can be instructive to inquire how various community groups, from business firms to cities to nations, learn. See Christensen and Hooker (2000a,b), Farrell and Hooker (2007a,b, 2009), Hooker (2009). Entities with a distinctive wholeness, individuality and perspective in the world, whose activities are willful, anticipative, deliberate, adaptive and normatively self-evaluated, are properly treated as genuine agents; and when their internal states and dispositions unfold historically, they are properly treated as full individuals; autonomous systems are inherently all of those things. See Hooker (2009), Skewes and Hooker (2009).

of a physical layer having functional roles necessary for system functioning. Thus while a crowd self-assembled at a free concert has a natural boundary (current behaviour influenced by direct sensory interaction with staged behaviour), the boundary carries out no function necessary for the concert itself; by contrast, a ticketed concert in a concert hall has as its boundary the physical barrier of the concert hall, forcing the interactive crowd boundary into conformity with it, but this boundary also has the necessary economic function of supporting the performance by discriminating audience access on the basis of payment.

The basic point of functionally active boundaries (and of some others) is to sustain an asymmetry (thermodynamic and organisational) between interior and exterior, by excluding some class of stimuli perturbing to internal system functioning (e.g. carapaces excluding blows, epidermises excluding toxins) and in more complex cases by gating the system (selectively admitting and expelling various material parts of the system, e.g. food in/ wastes out). In the simpler exclusion-only cases the boundary can be used to cleanly identify the system in space and time as what is physically internal to it. But it is surprising how quickly gating sets in; e.g. even the engine casing not only performs several different exclusionary functions (gas containment, vibration and shock resistence) but also fuel and exhaust gating. And once gating occurs identity typically cannot so simply be expressed. In the ticketed concert hall, e.g., audience and players may be let out (e.g. to toilet) and return yet remain part of the concert, even play key roles in it.

The ticketing barrier and engine casing gates roughly correspond to the ion gates a single cell has in its walls that selectively intake nutrients and expel wastes, but while there are only a few essential functions played by the former barriers, there are many, many more functional roles played by the cell membrane in maintaining cellular functioning. To this end the cell membrane contains many integral membrane proteins that serve specialised roles in membrane functioning, representing a huge increase in its functionality but also in its internal boundary membrane organisation to sustain those functions. In addition, the boundary itself has to be regenerated as part of the intra-cellular functions that boundary itself helps to facilitate. In biology functional boundaries become ubiquitous, diverse and complicated, ranging from those for chromosomes (nuclear membranes) and cells (membranes and walls), through those for tissues and organs (mesenteries and connective tissues), to those for multi-cellular organisms (epidermises), and finally(?) those for multi-organism groupings, e.g. bacterial colonies, slime moulds, termite nests, human clubs, cities and territories. Across this range boundaries vary greatly in their structure and properties, and also vary increasingly within each category (e.g. the large differences between moulds, fungi and animals within multicellular organisms, and between clubs, cities and territories).

Boundaries represent constraints. The point of the theatre boundary is to constrain entry of persons and exit of sights and sounds and structure communication to others in these terms. These features are true of all boundaries, even permeable ones. (Otherwise there is no identifiable boundary function.) However, in simple cases like the theatre or engine casing this boundary subsists in itself, its strong internal constraints allowing it to perform its functions largely in passive independence from the goings on in its interior. This has been typical of engineered boundaries. But now there is an increasing move toward more interactive boundaries, windows that tint or have covers that close in response to sunlight, doors that open automatically to select ID-scanned entrants, etc., and soon membranes that respond electrically, thermally and colourfully to various internal signals, and so on. This means that constraints themselves have to shift as a function of other processes occurring in the system, and thus that the system has to do work on its constraints. (Neither is a surprise, in highly and rapidly adaptive systems like the brain we face subtly interrelated constraint organisation that shifts on time scales from seconds to decades—cf. Christensen and Hooker (2000b), note 14 and text.) The point here is that as the boundary takes on increased functionality its internal constraints have to become both more organised and more subtly interconnected to processes and constraints internal to the system (but external to the boundary) for both functional responsiveness and energy. By the time we add the biological requirement that the internal system processes and constraints regenerate the boundary, including its constraint structure, we arrive at a very subtly organised articulation of constraints indeed.

This completes the review of fundamental complex systems properties and it makes the case for the central roles of constraints in characterising them. It also demonstrates the diversity of properties characteristic of complex systems and the internal complexity of many of them. This shows the insufficiency of any of the commoner simple ideas of complexity—number of degrees of freedom (component numbers), algorithmic incompressibility, levels,. . .—to capture the notion of complexity by themselves. To approach a characterisation, first omit all epistemic notions, e.g. those appealing to intelligibility and surprise, and all 'external' notions like controllability, as ultimately derivative considerations. Then at the least complexity looks to have five quasi-independent dimensions to it: cardinality (component numbers), non-linearity (of interaction dynamics), intermediate orderedness (algorithmic compressibility), nested organisation (organisational depth) and global organisation. I do not know of any one measure that can capture all these dimensions and I do not think anything less rich is adequate to the forms complexity can take. I do suggest that insight into the nature and forms of constraints will be important to gaining any deeper insight into the matter.

3 Challenges (A Tentative Exploration)

The explosion of complex systems concepts, models and methods across all the sciences and public policy over the past 30 years poses many theoretical and practical challenges to contemporary practices. The first science-wide review of the foundational challenges is found in Hooker's *Philosophy of Complex Systems*, Hooker (2011a), a collection of 27 essays by leading scientists and philosophers spanning the sciences and public policy; readers are directed there for further exploration. In this section three of these challenges, deriving importantly from the roles of constraints, will be briefly discussed to illustrate the range and importance of constraint-driven challenges.[24]

3.1 Organisation

Global constraints pose two distinctive sorts of challenge to conceptualising complexity in modern science: they introduce the concept of organisation as a key aspect of complexity, a concept that currently has no satisfying measure, and they challenge the representation of global constraints in mathematical dynamics.

The internal articulation of distinct components so as to be globally functionally coherent, found in the motor vehicle engine, provides a paradigm of what is needed to capture organisation proper.[25] Machines and living things are organised because their parts are relatively unique and each part plays distinctive and essential roles in the whole. In this sense, each global functional constraint requires an underlying organisation of dynamical constraints and processes to realise them. The set of coordinated constraints required to realise organisation

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[24] The first and last discussions that follow are adapted, with modification and improvement, from Hooker (2011b,d) and the reader is referred there for further discussion.

[25] There is a wider colloquial usage in which being organised merely means being appropriately prepared, whether or not that preparation involves any significant organisation in the narrower sense in the text.

is less ordered than those of a crystal, which is simply and uniformly ordered, but much more ordered than the null coordination within a gas. Thus orderedness, measured by (the inverse of) algorithmic complexity, cannot capture organisation. We approach more closely the notion of organisation by considering the order of the relationships within the constraint coordination, where a 2nd order relation is a relation among relations, and so on. Then an organisation is characterised by (relatively) high order relations that involve many nestings of relations within relations, that is, of nested correlations within correlations. For example, a car engines pistons and valves are internally correlated and these correlation relations are nested within correlations between cylinder positions and the fuel injection system, etc. A system's organisational depth is measured by the degree of nesting of sub-ordering relations within its global ordering relation (cf. cells within organs within bodies within communities). Living systems especially are deeply organised. However, organisational depth also does not fully capture complexity.[26] Thus the coordination of constraints that marks global functional organisation is somewhat subtler than we can at present fully characterise.

Neither can we properly represent it mathematically. Dynamics is presently represented mathematically as a differentiable flow on a structured manifold, e.g. Classical mechanics can be represented as a flow or field of trajectories in space determined by the system Lagrangian, a representation of energy, the motion represented by differential equations. These modelling resources, powerful though they are for modelling the energetics of processes, do not explicitly describe the physical organisation of the system. For instance, a metabolic cycle and a pendulum may be modelled as equivalent dynamical oscillators. In a phase space only the global dynamical states and their time evolution along a system trajectory of the overall flow, is specified, not the organised processes that produce the dynamics. Note that any differential equation already compresses constraint and interaction information into a flow and in that sense suppresses the explicit details of the interactions and constraints. The globalness and organisation of the constraints then pose a further, particular problem to representation. And the reverse engineering problem of specifying organisation (as distinct from order) from dynamics is currently unsolvable.[27] In this sense our current fundamental representation of dynamics cannot capture organised constraint coordination, and hence also not the mechanisms that realise them.[28]

Recall from earlier discussion that organisation of constraints (i) stands at the heart of all biological entities, (ii) extends to the subtle organisation underpinning functional fluency and

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[26] Because, e.g., it does not capture the distinctiveness of nested relations (cf. note 28) and the top-down constraints that modulate them and, in dropping the algorithmic conception, it loses 'horizontal' relational complexity. Gell-Mann Gell-Mann (1994) discusses effective complexity and logical depth (see Bennett 1985, 1992) and Type 2 theories (Marr 1982), as other possibilities for measuring organised complexity, but neither is satisfactory for various reasons he notices—fundamentally for the above reasons. For general discussion of these issues see Collier and Hooker (1999), sections III and VI.

[27] Current reverse engineering programmes presume simple system structures that do not include global constraints and organisation, cf. Bongard and Lipson (2007), Tegnér et al. (2003). This is the counterpart to the severe constraints on programmes for the extraction of causal order in systems, that presume that not even simple feedback loops are present (cf. Shalizi 2006, 2.2.1). However work continues on increasing the discriminatory capacities of reverse engineering—for an interesting development see Schmidt and Lipson (2009)—and it is too soon to pronounce on its limits.

[28] It will of course often be possible to make scientific headway by identifying organised biochemical mechanisms that contribute to realising such constraints, with the Krebs cycle a powerful case in point (see Bechtel and Abrahamsen 2011). And since each individual interaction is derived from basic physical interactions, each can be represented dynamically and the set of them can be modelled as a set of coupled integro-differential equations. But what cannot be represented dynamically, it seems, is their particular nested cyclical organisation; the coupled equations represent any organised collection of interactions whatsoever so long as its net connectivities are represented by these couplings.

effectiveness, e.g. in multi-tasking and multi-plexing, and (iii) underlies virtually all social and much ecological organisation. Thus these limitations form a major foundational challenge to the science of complex systems. Moreover, that challenge extends in concrete form to robotics and other so-called intelligent systems: since none of these as yet has a substantial autonomous organisational basis for its capacities, its intelligences, however skilled in specialised ways they may be, remain unintegrated into their root agency capacities, precluding them from solving the fundamental class of open problems and from open cognitive development more generally.[29]

3.2 Boundaries

The challenge here is to general ignorance: I have been able to locate little general knowledge, either theoretical or practical, of when functionally active boundaries are necessary, what their functional roles should be, and of how their constraints and dynamical interactions should be organised to achieve their roles. Despite a slow increase in the interactiveness of some engineered boundaries, we have not yet seen boundaries in engineering of the biological sort and likely will not see them until bioengineering, barely begun, matures. (Most or all current engineering experiments in self-assembly or regenerative re-assembly of which I am aware don't include any significant boundary layer.) However it is the biological cases of organised coordination of boundary constraints in the service of functionality we need to understand if we are to understand the possibilities for complex system function that boundaries offer and biology already exploits. Such understanding is also required to appreciate when, why and how super-systems can form from interacting systems, for instance the differences between the agglomeration and fruiting stages of slime mould assembly, or between interpenetrating DNA-sharing primitive plants and modern plants, or between an ancient walled city and a modern city, and so on.

Conversely, we need to understand when, why and how the biological boundaries we do possess as mammals permit us to construct social organisations with somewhat more open boundaries. Modern cities and regions, e.g., can show significant autonomy, yet have boundaries thoroughly open to all kinds of traffic across them. Threats produce toughened boundaries in response, from quarantining SARS victims (Singapore) to battlefield barriers, but modern cities and regions are in general far more permeable than were ancient walled cities and closed colonies. These modern, more open structures dictate that autonomous systems

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[29] See references, note 23. Robotics uses a very limited formal notion of autonomy (something like invariant dynamical form) and limited performance criteria (typically confined to a single task) and an equally limited satisfaction method. There has recently emerged an embodied functionality movement within robotics (see e.g. Nolfi 2011; Pfeiffer and Bongard 2007) where cognitive organisation is strongly shaped by the dynamics of body and environment, in ways that you would expect from an autonomy, interactive perspective. This represents a vast improvement over the computer-on-a-machine approach that had previously dominated. However it is as yet very far from even incorporating normative signals into the body coherence of robots, let alone the complexity required for self-regeneration and the capacity for fluid management of multi-dimensional environmental and internal interaction processes in relation to that. While studies such as that by Nolfi (above) have made progress on the fluid management of environmental interaction, these are still primitive devices when it comes to management of activity in relation to norm-derived goals. The problem in artificial life is still further from solution, since formal reproduction is not regenerative and is not the core of metabolism and thus not the key to metabolism-based action norms. See Moreno and Ruiz-Mirazo (1999); Moreno et al. (2011), cf. Christensen and Hooker (2002, 2004). There is an associated need to bring work on self-assembling, self-repairing robots (e.g. Groß and Dorigo 2007, 2008, http://www.swarmanoid.org/index.php) into relation with attempts to develop artificial autonomous systems where modelling even very elementary cells that are dynamically stable and thermodynamically coherent is proving difficult (e.g. Gánti 2003; Ruiz-mirazo and Moreno 2004; Szathmáry 2005; Barandiaran and Ruiz-mirazo 2008, http://www.ees.lanl.gov/protocells).

are ultimately identified, not by having a specific continuous physical boundary layer, but with the scope of their autonomous self-regulation, that is, with the scope of their organised constraints. Inside = inside the scope of self-regulatory constraint, this is the locus of the key asymmetry between system and environment and what is common across all biological systems. Generalising, the 'self' of self-organisation is what is within the scope of the relevant constraint-forming dynamics.[30] The switch from grounding identity and internality on boundary possession then opens up exploration of the kinds of boundaries that may facilitate autonomous self-regulation in various circumstances. We need to understand where and how autonomy might be expressed within porous boundaries, or perhaps relinquished because reliance on prior functional boundaries can be made to suffice.[31]

Few researchers seem to have addressed these issues. Among the few is Alan Raynor in his provocative *Degrees of Freedom: Living in Dynamic Boundaries*, Raynor (1977).[32] He contends that the general function of boundaries is to co-adapt internal and external environments, and perhaps the boundary itself also along the way (the cell lipid membrane preceded the cell) and that to this end there are 3 universal functional properties of boundaries: permeability, deformability and continuity, that may be realised in varying degrees, sometimes across many intra-boundary layers. This much is not nothing, but may be all the general boundary theory that can currently be achieved. At the least it reminds us of two important factors, internal/external co-evolution and boundary evolution, that should be included in a proper dynamical conception of evolution. But considering the ubiquity of boundaries and the crucial roles they often play in system dynamics—by applying various forms of constraint—we must hope for more general understanding of this subtlest of applications of organised constraints if we are to develop general foundations for a theory of complex systems and valid methods to scientifically investigate them.

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[30] This in turn provides the foundation for the principle that all systems processes must ultimately be enabled by, and only by, interactions and constraints available internally to the system. See e.g. Bickhard (1993), Christensen and Hooker (2000b), note 4 and text for one among several earlier versions by others, including Newman (1970) (Newman 2011a,b—private communication). This principle removes evolutionary views of proper function, source-based constructs of signal meaning, and so on.

[31] Even physiologically, multi-cellular creatures have wider commerce through their epidermic boundaries than do simpler creatures; humans, e.g., do not make nine essential amino acids but import them through eating plants. This issue arises within a larger trend associated with multicellular organisms. Overall, the effect of multi-cellular evolution has been to expand the capacity for interaction with the environment, including both anticipating environmental courses of action and acting to modify the environment to shape its selection pressures. Multi-cellulars differ in at least three important respects from single cells: they have (i) increased substitution of environmental construction for internal construction (e.g. carnivores intake complex molecules, humans rely on environmental production of many essential amino acids), (ii) increased self-regulation of their food acquisition and damage avoidance (e.g. rapid or prolonged migration to track food resources, hiding or hole construction to escape predators) and (iii) increased capacity to self-regulate the modification of metabolism to suit both temporary activity (e.g. heart rate and blood re-direction for running) and permanent change (e.g. callousing, neuro-muscular compensation for injury). Underlying these is a fourth, more basic, way in which they differ: (iv) they have acquired the capacity to communally regulate the birth, specialisation and death of their members (cells). While in neurally more complex species they can show a myriad of forms, every viable community, including human communities, must acquire some form of these latter capacities. (Thanks to Alvaro Moreno for this insight.) Over the last century as human societies have become more developed, they seem to be experimenting with internal regulation of birth (aka the demographic transition), and specialisation (aka education) while decreasing regulation of death (no death penalty, voluntary euthanasia, + some regulation of war). Such shifts require increasing internal self-regulatory capacities.

[32] Discussions with Raynor, and reading his work, form an important part of my appreciation of dynamic boundaries, and I thank him for them. But there are also significant ways in which I depart from his 'boundary-centric' approach, perhaps best illustrated in note 31 and text.

3.3 Dynamical Representation

Is every behaviour of complex systems equally representable within classical dynamics? The argument from basic interaction answers yes: all of the basic interactions among components, including the constraint forming components, are from classical dynamics (gravity and electro-magnetism) so the behaviour of the systems themselves must be representable in classical dynamics.[33] In fact, the answer is no; the argument from basic interaction is invalid and its failure turns on the kinds of constraints involved. This failure of dynamical representation constitutes the most fundamental challenge of complex systems to standard dynamics.

It is not hard to locate the fault line in the argument from basic interaction: it assumes that once the basic interactions are given, all other interactions will have thereby been fully captured. But some complex systems defeat this assumption. The difficulties come in two forms.

First, there is the problem of the representation of integrated multi-phasic systems. In Sect. 1 it was pointed out that most biological systems have simultaneously operating constraints of many different kinds, mechanical and chemical, solid and liquid, and so forth, operating in an equal diversity of ways: quasi-equilibrium, non-equilibrium oscillatory, phasic, etc. How is this diversity to be captured within an integrated dynamical representation? Solid and liquid models, e.g., are very different in form, any direct conjunction of the two will simply produce a contradictory model. Sometimes a hybrid model can be constructed by cleverly shaping a coherent amalgam of a very few selected principles from each that nonetheless capture the phenomenal domain of interest, but this is rare and certainly not to be presumed. Combining mechanical and chemical processes faces similar (if perhaps less severe) problems. In fact such problems can arise within a single kind of dynamical domain. In the case of constrained viscous fluid flow, e.g. through a pipe, the dynamical determinants of motion near a wall are very different to those near the centre and each requires a distinctive model incompatible with that for the other (Rueger 2005). These dynamical representations cannot be unified. For systems like organisms and cities there is no workable option but to investigate partial aspects of them one at a time, often of necessity using knowingly idealised models, e.g. constrained focal-process + process-perturbation and constrained focal-level plus cross-level interaction models.[34] Were this the only integration problem it could be seen as an unavoidable, but ultimately pragmatic, consequence of the limits of mathematical representation, at least in the hands of finite agents; but it is not the only problem.

Second, for a certain class of constraints, there is a further, deeper problem for dynamical representation. To understand what is involved it is necessary to return to the fundamentals of classical dynamics. There is a generalised analytical framework for classical dynamical analysis – the Lagrangian/Hamiltonian formalism – that directly or approximately covers a wide range of cases and serves as the core of analytical classical dynamics.[35] In this scheme, recall, basic interaction dynamics and constraint forces combine to produce the system dynamics.

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[33] In fact the challenge to this conclusion will not depend on whether the basic interactions are classical, relativistic or quantal, while most examples are classical.

[34] The unified representation problem arises when each model is a degenerate idealisation of the full interaction dynamical flow equations—that is, one that collapses out structure that cannot be regained by any subsequent conjunction of additional detail, as with the two fluid flow approximation models. The one system may have many different, mutually incompatible, degenerately idealised dynamical models (Hooker 2011c, on degenerate idealisation see also Hooker 1994, 2004). However, where the full flow equations are not workable, in principle or in practice, there is no choice but to use degenerately idealised models, and this applies still more forcefully to biological systems.

[35] For a nice introduction see Butterfield (2004a,b). Classic texts here are Goldstein (1950), Arnold (1978). Butterfield (2004a), note 6 offers a brief guide to others. Bloch (2003) offers an introduction to the treatment

External constraints typically introduce unknown forces into the dynamics, so that a determinate Newtonian dynamics cannot be specified, and they result in interdependencies among the intrinsic dynamical variables that have to be accommodated, so that an unambiguous representation of the dynamical possibilities cannot be formulated. To resolve these problems it is sufficient, and arguably necessary, to restrict consideration to those systems where the external constraint forces act orthogonally to all allowed system motions,[36] so that the system does no work against external constraints (constraint force orthogonality). This defines a constraint (hyper) surface in the system configuration space to which the constraint forces are everywhere perpendicular (orthogonal). Thus motion on this surface is effectively constraint-free – this is expressed in D'Alembert's principle. If in addition the external constraints are holonomic – literally: express a whole or single law[37] – then the system dynamics may be re-formulated on their D'Alembertian constraint surface in terms of new generalised, independent variables, the dynamics now having the form of a free (unconstrained) system. Lagrange equations of motion can then be formulated for the system. This resolves the variable interdependency problem introduced by constraints. The method of Lagrange multipliers then formulates D'Alembert's extremal least action principle and permits solving the system dynamics (that is, specifying the action geodesics) on the constraint surface, without knowing the external constraint forces. Rather, once the dynamics is known, the external constraint forces can be reconstructed as the forces they need to be to maintain the external constraints during the system motion. This resolves the problem of their being initially unknown.

Many complex systems do work on their constraints, physically altering them over time. Examples include (i) a river altering its own banks, an accumulative process where the current constraints (banks) are a function of the history of past flows (currents), (ii) intra-cellular biochemical reaction processes where molecular structures constraining some processes are the products of other processes and vice versa; (iii) any self-organisation where the constraint formed becomes an external constraint for subsequent processes (Bénard cell and iron bar formation, etc.). In all these systems constraint orthogonality fails. With this failure the most basic precondition for achieving the core analytic construction fails. There is then no general, analytical mathematical formalism available for dynamical behaviour. The incapacity to construct conditions where D'Alembert's principle holds undermines the applicability of the very variational apparatus that we take to underlie all fundamental dynamics. In this way,

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Footnote 35 continued
of D'Alembertian but non-holonomic constraints, see also Flannery (2005). While, strictly, Newton's Laws themselves can in principle be applied to any system, with holonomic or non-holonomic, conservative or non-conservative constraints, the Lagrangian analytical apparatus not only captures the core cases and provides powerful analytical tools for analysing dynamics, it is also the basis for treating continua and force fields, especially the electromagnetic field, and its founding variational principles form the basis for the successful generalisations of mechanics to relativistic and quantum mechanical forms.

[36] These are the 'virtual' displacements of a system, as opposed to actual displacements over some small time interval occurring under the influence of the intrinsic forces as well.

[37] Holonomic constraints may be written as some function of the space-time geometry in which the system moves (note 9). Specifically, they satisfy an equation of the form $f(r_1, r_2, \ldots r_n, t) = 0$, where the r_i are system coordinates and t is time. This expresses the effect of the constraint forces while not specifying the forces themselves. (The forces are often known only after the main problem is solved.) While smooth (frictionless) sliding under gravity on a sloping plane is a case of holonomic constraint, a spherical bead rolling smoothly on the outside of a cylinder is not because the constraint alters its basic character when the bead falls off. Essentially, for the constraints to be holonomic means that they may be expressed purely geometrically, so that they are independent of the behaviour of the system. Independence fails in the case of the bead on the cylinder, there is a change of constraints at a space-time location determined by the bead's motion. (Note that the reverse relation does not hold, e.g. though independent of system behaviour, containment walls do not form holonomic constraints.)

complex systems challenge the reach of our deepest analytical understanding of dynamics and thus present a fundamental dilemma about how to approach dynamics: retain the present approach and exclude complex systems or search for some new, more generous foundations for dynamics.[38]

The response of those favouring the argument from basic interaction (also the textbook response, when there is one) is to argue as follows: the system can always be expanded so as to include the matter that is the source of the constraint forces and when that is done we discover only further basic interactions of the usual kinds, so the enlarged system undergoes standard constraint-free motion and the problem vanishes. If we cannot solve these systems then it is simply because there are too many components involved, a pragmatic rather than a principled difficulty. This appears persuasive at first blush (e.g. it satisfied no less than Goldstein 1950, p.14), constraints really do arise from basic interactions, otherwise we could not understand dynamically their transformation by the system. But we should look further. Recall that the argument as it stands is at least incomplete: the leap to the conclusion again requires assuming that once the basic interactions are given, all other interactions will have thereby been fully captured, begging the point at issue. It should be considered that in all the cases where there are dynamical constraints that do have work done on them in the course of the dynamics this work can be objectively measured, so it cannot simply be claimed that a constraint-free representation is always available, it has to be shown.

As the point was put in Sect. 1 discussing Bénard cell formation, the constraints don't disappear, they must re-appear internally. Nor therefore is it enough to point out that the work done on constraints must ultimately itself be constituted in basic dynamical processes; this is true, but not the issue. The issue is the reality of supra-basic dynamical formations that can filter other motions.[39] The reality of these is itself guaranteed by the constraint-forming basic interactions, wherever a suitable invariant level suffices explanatorily.[40] Those who deny such compound dynamical entities owe it to us to show how to both faithfully represent the internal presence of the constraints—at all, and if admitted, show the work done on and

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[38] There is also a whiff of scandal here as well, namely the scandal, unfortunately increasingly common, of dynamics textbooks simply ignoring these deep problems, or implying that there is only a pragmatic issue of mathematical resources involved.

[39] Bishop expresses this as follows: The properties and behaviors of a system at a particular level (including its laws) offer necessary but not sufficient conditions for the properties and behaviors at a higher level. He calls this contextual emergence. (See Bishop 2005, 2008, 2011). This is apt, since it points to an irreducible wholeness (emergence) under specific dynamical conditions. Hooker (2004) makes the complementary point that each specific reduction of function to dynamics in complex systems requires a dynamically specified context, so that we also only have contextual reduction—and, since the context must specify appropriate constraints, it must include dynamically irreducible features, so that reduction and emergence are intertwined.

[40] Bénard cell formation involves only behavioural change (in a fluid), and so is clearly a supra-basic dynamical formation over fluid elements, a dynamical wholeness, not any new fundamental kind of entity. However typical phase transitions show qualitative as well as behavioural changes, e.g., from solid to fluid; but these too are representable as supra-basic dynamical formations if there is a component level at which the components are invariant through the transition, e.g. molecules for most phase transitions. This idea then extends to those transitions that also involve the erstwhile basic components also changing qualitatively, e.g. undergoing chemical change, becoming ionised (in transitions to plasma) or undergoing nuclear change (e.g. in gravitational condensations), so long as the level of (respectively) atoms, electrons, and nucleons suffices explanatorily (other than for Lagrangian descriptions of the transitions). If so, all these changes can be understood as purely dynamical formations over invariant basic components under constraint changes. If not, then we have to contemplate a mechanics of basic transformation, perhaps equipped with quantum-like creation and annihilation processes, something also beyond Lagrange theory. Thus getting the dynamical specifications right is what is ultimately important. Thanks to Gil Costa Santos now, and Parker English in 1973, for pointing to these telling metaphysical issues.

by them—and still achieve an analytical Lagrangian model.[41] This is prima facie not possible. Constraint forming systems, like the iron bar from cooling, create similar challenges to Lagrangian representation. All these challenges may be restated as follows: since precisely in all such processes the system changes dynamical form, hence would change Lagrangian form, it is unclear how the Lagrangian apparatus in itself could accommodate that requirement. In short, it is difficult to see how systems showing these phenomena could be reduced to presenting merely pragmatic barriers to standard knowledge of solutions.[42]

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References

Arnold, V. I. (1978). *Mathematical methods of classical mechanics*. Berlin: Springer.

Barandiaran, X., & Ruiz-mirazo, K. (Eds.) (2008). Modelling autonomy: Simulating the essence of life and cognition. *BioSystems, 91*(2).

Bechtel, W. (2007). Biological mechanisms: Organised to maintain autonomy. In F. Boogerd, F. Bruggeman, J.-H. Hofmeyr, & H. V. Westerhoff (Eds.), *Systems biology: Philosophical foundations* (pp. 269–302). Amsterdam: Elsevier.

Bechtel, W., & Abrahamsen, A. (2011). Complex biological mechanisms: Cyclic, oscillatory and autonomous. In C. A. Hooker (Ed.) Philosophy of complex systems. *Handbook of the Philosophy of Science* (Vol. 10, pp. 259–288). Amsterdam: North Holland/Elsevier.

Bennett, C. (1985). Dissipation, information, computational complexity and the definition of organization. In D. Pines, *Emerging syntheses in science, proceedings of the founding workshops of the santa fe institute*. Redwood California: Addison Wesley.

Bennett, C. (1992). Logical depth and other algorithmically defined properties of finite objects. In *Proceedings, workshop on physics and computation*, IEEE, pp. 75–77.

Bickhard, M. H. (1993). Representational content in humans and machines. *Experimental and Theoretical Artificial Intelligence, 5*, 285–333.

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[41] Of course we can construct little Newtonian models of work done on and by some specific constraints, e.g. of flowing water eroding a bank, and gesture at others. This too is true but not germane. The requirement is to show how to coherently incorporate these into a single Lagrangian model of the whole system, as the pragmatic claims presume. Nor is it germane to point out that D'Alembert's principle requires only that the *net* work done in virtual displacements is zero (e.g. Subhankar and Shamanna 2006) and that in a basic dynamical model of system+constraints this is satisfied if Newton's Third Law operates to ensure balanced forces by the system on the constraints and by the constraints on the system, since the point is to capture the internal dynamical reality of the filtering constraints. Similarly, we can try to approximate each real constraint as a time-slice sequence of holonomc constraints for purposes of analysis, but this is again just to concede the point at issue. Finally, controlling systems with non-holonomic constraints does not by itself count either, since this can be achieved without needing a full analytical model of the dynamics (e.g. through local curve tracing, cf. Bloch 2003).

[42] There are further technical issues listed as outstanding in Hooker (2011d), most importantly the extent to which the applicability of the analytical Lagrangian model can be pushed into the domain of non-holonomic constraints. The extension can be made specifically to semi-holonomic and exact linear constraints Flannery (2005) and those meeting various other convenient (but not necessarily well-grounded) conditions, like the Chetaev condition, but this did not leave a clear picture of the situation. Recently Flannery (2011a), in a helpfully clear historical review (cf. Soltakhanov et al. 2009) and new generalised analysis, has succeeded in extending the analytical Lagrangian model to all ideal non-holonomic constraints, i.e. to all those constraints that act orthogonally to the motions and so have no work done on them during motion. (See also Flannery 2011b). For the reasons noted in the text, it remains that this analysis cannot be extended to non-ideal constraints, i.e. to systems that form constraints or do work on their constraints. Another outstanding issue was when and why the consequent extremal action paths (all Hamilton's Principle requires) are least action paths. This has been interestingly settled for one class of cases in Gray and Taylor (2007), but cf. Dewar et al. (2011) for some of the complications involved elsewhere.

Bishop, R. C. (2005). Patching physics and chemistry together. *Philosophy of Science, 72*, 710–722.

Bishop, R. C. (2008). Downward causation in fluid convection. *Synthese, 160*, 229–248.

Bishop, R. C. (2011). Metaphysical and epistemological issues in complex systems. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 107–138). Amsterdam: North Holland/Elsevier.

Bloch, A. M. (2003). *Nonholonomic mechanics and control*. Springer, New York, See http://www.cds.caltech.edu/mechanics_and_control/about_this_book/ for corrections and additions, incl. internet supplement.

Bongard, J., & Lipson, H. (2007). Automated reverse engineering of non-linear dynamical systems. *Proceedings of the National Academy of Science, 104*(24), 9943–9948.

Butterfield, J. (2004a) Between laws and models: Some philosophical morals of Lagrangian mechanics. Available at http://philsci-archive.pitt.edu/archive/00001937/. Accessed Apr 2009.

Butterfield, J. (2004b). On Hamilton-Jacobi theory as a classical root of quantum theory. In A. C. Elitzur, S. Dolev, & N. Kolenda (Eds.), *Quo Vadis quantum mechanics? Possible developments in quantum theory in the 21st century*. New York, Springer. Available at http://philsci-archive.pitt.edu/archive/00001193/. Accessed Apr 2009.

Christensen, W. D., & Hooker, C. A. (2000a). Organised interactive construction: The nature of autonomy and the emergence of intelligence. In: A. Etxebberia, A. Moreno, & J. Umerez (Eds.), *The contribution of artificial life and the sciences of complexity to the understanding of autonomous systems. Communication and Cognition*, Special Edn (Vol. 17, No. 3–4, pp. 133–158).

Christensen, W., & Hooker, C. A. (2000b). An interactivist-constructivist approach to intelligence: Self-directed anticipative learning. *Philosophical Psychology, 13*, 5–45.

Christensen, W. D., & Hooker, C. A. (2002). Self-directed agents. In J. MacIntosh (Ed.), *Naturalism, evolution and intentionality. Ottawa, Canadian Journal of Philosophy*, Special Supplementary Vol. 27, pp. 19–52.

Christensen, W. D., & Hooker, C. A. (2004). Representation and the meaning of life. In H. Clapin, P. Staines, & P. Slezak (Eds.), *Representation in mind: New approaches to mental representation* (pp. 41–69). Sydney: Elsevier.

Clarke, D., & Proença, J. (2009). Coordination via interaction constraints I: Local logic. In F. Bonchi, D. Grohmann, & P. Spoletini, E. Tuosto (Eds.), *ICE'09 structured interactions*, EPTCS, 12, pp. 17–39. doi:10.4204/EPTCS.12.2.

Coleman, R. A., & Korté, H. (1999). Geometry and forces in relativistic and pre-relativistic theories. *Foundations of Physics Letters, 12*(2), 147–163. doi:10.1023/A:1021609022945.

Collier, J. D., & Hooker, C. A. (1999). Complex organised dynamical systems. *Open Systems and Information Dynamics, 6*, 241–302.

Dewar, R. L., Hudson, S. R., & Gibson, A. M. Action-gradient-minimizing pseudo-orbits and almost-invariant tori (preprint, submitted).

Emmeche, C., Koppe, S., & Stjernfelt, F. (2000). Levels, emergence, and three versions of downward causation. In P. Anderson, C. Emmeche, N. Finnemann, & P. Christiansen (Eds.), *Downward causation: Minds, bodies, and matter* (pp. 13–34). Aarhus: Aarhus University Press.

Farrell, R., & Hooker, C. A. (2007a). Applying self-directed anticipative learning to science: agency and the interactive exploration of possibility space in ape language research. *Perspectives on Science, 15*(1), 86–123.

Farrell, R., & Hooker, C. A. (2007b). Applying self-directed anticipative learning to science: Learning how to learn across revolutions. *Perspectives on Science, 15*(2), 220–253.

Farrell, R, & Hooker, C. A. (2009). Applying self-directed anticipative learning to science: identifying error and severe testing through interactive exploration of possibility. *Foundations of Science, 14*(4), 249–271.

Flannery, M. R. (2005). The enigma of nonholonomic constraints. *American Journal of Physics, 73*(3), 265–272. Available at www.physics.gatech.edu/people/faculty/flannery/publications/AJP73_March2005_265-272.pdf

Flannery, M. R. (2011a). D'Alembert–Lagrange analytical dynamics for nonholonomic systems. *Journal of Mathematical Physics, 52*(3), 032705. doi:10.1063/1.3559128.

Flannery, M. R. (2011b). The elusive d'Alembert-Lagrange dynamics of nonholonomic systems. *American Journal of Physics, 79*(9), 932–944.

Gánti, T. (2003). *The principles of life*. New York: Oxford University Press.

Gell-Mann, M. (1994). *The Quark and the Jaguar: Adventures in the simple and the complex*. New York: Henry Holt.

Goldstein, H. (1950). *Classical mechanics*. Reading, Mass: Addison-Wesley.

Gray, C. G., & Taylor, E. F. (2007). When action is not least. *American Journal of Physics, 75*(5), 434–458.

Groß, R., & Dorigo, M. (2007). Fifty years of self-assembly experimentation. In W.-M. Shen, H. Lipson, K. Stoy, & M. Yim, *Proceedings of the workshop on self-reconfigurable robots/systems and applications*. USA: USC Information Science Institute.

Groß, R., & Dorigo, M. (2008). Self-assembly at the macroscopic scale. *Proceedings of IEEE, 96*(9), 1490–1508.

Hooker, C. A. (1994). Idealisation, naturalism, and rationality: Some lessons from minimal rationality. *Synthese, 99*, 181–231.

Hooker, C. A. (2004). Asymptotics, reduction and emergence. *British Journal for the Philosophy of Science, 55*, 435–479.

Hooker, C. A. (2009). Interaction and bio-cognitive order. *Synthese, 166*(3), 513–546.

Hooker, C. A. (Ed.). (2011a). *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10). Amsterdam: North Holland/Elsevier.

Hooker, C. A. (2011b). Introduction to philosophy of complex systems. Part A: Towards framing philosophy of complex systems. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 3–92). Amsterdam: North Holland/Elsevier.

Hooker, C. A. (2011c). Introduction to philosophy of complex systems. Part B: Aninitial scientific paradigm + philosophy of science for complex systems. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 843–912). Amsterdam: North Holland/Elsevier.

Hooker, C. A. (2011d). Conceptualising reduction, emergence and self-organisation in complex dynamical systems. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 197–224). Amsterdam: North Holland/Elsevier.

Kaufman, S. (2000). *Investigations*. New York: Oxford University Press.

Mackworth, A. K. (2009). *Agents bodies constraints dynamics evolution* (pp. 7–28). Spring: AI Magazine.

Marr, D. (1982). *Vision*. Cambridge, Mass: MIT Press.

Maynard-Smith, J., & Szathmary, E. (1995). *The major transitions in evolution*. New York: Freeman.

Moreno, A. (2007). A systemic approach to the origin of biological organization. In F. Boogerd, F. Bruggeman, J.-H. Hofmeyr, & H. V. Westerhoff (Eds.), *Systems biology: Philosophical foundations* (pp. 243–268). Amsterdam: Elsevier.

Moreno, A., & Ruiz-Mirazo, K. (1999). Metabolism and the problem of its universalization. *Biosystems, 49*, 45–61.

Moreno, A., Ruiz-Mirazo, K., & Barandiaran, X. (2011). The impact of the paradigm of complexity on the foundational frameworks of biology and cognitive science. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 313–336). Amsterdam: North Holland/Elsevier.

Newman, S. A. (1970). Note on complex systems. *Journal of Theoretical Biology, 28*, 411–413.

Newman, S. A. (2011a). Complexity in organismal evolution. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 337–356). Amsterdam: North Holland/Elsevier.

Newman, S. A. (2011b). The animal egg as evolutionary innovation: A solution to the 'embryonic hourglass' puzzle. *Journal of Experimental Zoology, B: Molecular and Developmental Evolution, 316*(7), 467–483.

Newman, S. A., & Bhat, R. (2008). Dynamical patterning modules: Physico-genetic determinants of morphological development and evolution. Physical Biology, 5, 015008. Available at http://stacks.iop.org/PhysBio/5/015008.

Nolfi, S. (2011). Behavior and cognition as a complex adaptive system: Insights from robotic experiments. In C. A. Hooker (Ed.), *Philosophy of complex systems. Handbook of the philosophy of science* (Vol. 10, pp. 445–467). Amsterdam: North Holland/Elsevier.

O'Connor T., & Wong, H. Y. (2002) Emergence. Stanford Encyclopedia of Philosophy. http://plato.stanford.edu/entries/properties-emergent/

Pattee, H. H. (1971). Physical theories of biological co-ordination. *Quarterly Reviews of Biophysics, 4*, 255–276. Reprinted in M. Grene, E. Mendelsohn (Eds.), *Topics in the philosophy of biology*, Reidel, Amsterdam, 1976. doi:10.1017/S0033583500000640 (published online) 2009.

Pattee, H. H. (1973). The physical basis and origin of hierarchical control. In H. H. Pattee (Ed.), *Hierarchy theory: The challenge of complex systems* (pp. 71–108). New York: Geo. Braziller.

Pfeifer, R., & Bongard, J. C. (2007). *How the body shapes the way we think: A new view of intelligence*. Cambridge, Mass: MIT Press.

Raynor, A. (1977). *Degrees of freedom: Living in dynamic boundaries*. London: Imperial College Press.

Rueger, A. (2005). Perspectival models and theory unification. *British Journal for the Philosophy of Science, 56*(3), 579–594.

Ruiz-mirazo, K., & Moreno, A. (2004). Basic autonomy as a fundamental step in the synthesis of life. *Artificial Life, 10*(3), 235–259.

Shalizi, C. R. (2006). Methods and techniques of complex systems science: An overview. In T. Deisboeck, J. Kresh (Eds.), *Complex systems science in bio-medicine*. Springer, New York. Also available at http://arXiv.org/abs/nlin/0307015.

Schmidt, M., & Lipson, H. (2009). Distilling free form natural laws from experimental data. *Science, 324*(5923), 81–85.

Skewes, J., & Hooker, C. A. (2009). Bio-agency and the problem of action. *Biology and Philosophy, 24*(3), 283–300.

Subhankar R., & Shamanna, J. (2006) Understanding D'Alembert's principle: System of pendulums. Available at http://arxiv.org/PS_cache/physics/pdf/0606/0606010v2.pdf.

Soltakhanov, S. K., Yushkov, M. P., & Zegzhda, S. A. (2009). *Mechanics of non-holonomic systems*. Heidelberg: Springer.

Szathmáry, E (2005). Life: In search of the simplest cell. *Nature, 433*(3), 469–470.

Tegnér, J., Yeung, M. K. S., Hasty, J., & Collins, J. J. (2003). Reverse engineering gene networks: Integrating genetic perturbations with dynamical modeling. *Proceedings of the National Academy of Science, 100*(10), 5944–5949.

Turing A. (1952) The chemical basis for morphogenesis. *Philosophical transactions of the Royal Society of London, B: Biological Sciences, 237*(641), 37–72. Available at http://www.jstor.org/stable/92463.

Wackerbauer, R., Witt, A., Atmanspacher, H., Kurths, J., & Scheingraber, H. (1994). A comparative classification of complexity measures. *Chaos, Solitons & Fractals, 4*(1), 133–173.

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